Synopsis For Ph.D

TOPIC - "INVESTIGATIONS ON FIXED POINT THEOREMS IN BANACH AND OTHER SPACES".

1. INTRODUCTION

Investigations on fixed point theorems in Banach and other spaces have drawn much attention of several researchers and authors in recent years. This field has been proved to be a very useful tool for modern mathematics in present era. Which solves different diverse problems in Engineering, Biology [1], Economics [4], Social science[2] and Computer science [7]

S.Banach [3] established a given fixed point theorem for a contractive mapping in Complete metric space. The fixed point as observed by Banach is unique. one very important theorem in this direction is due to Kannan [13,14].

In 2007, Huang and Zhang [11] introduced the notion of Cone metric space replacing the set of real numbers by ordered Banach space and proved some fixed point theorems for functions Satisfying Contractive Conditions in this space. Kara Pinar [12] proved some fixed point theorem for self mapping satisfying some Contractive Conditions on a Cone Banach space.

Browder [5, 6], Goebel [8] and Kirk [15] have independently proved a fixed point theorem for non-expansive mapping defined on a closed bounded and Convex Subset of a uniformely Convex Banach space.

The Study of in wordmapping (Contraction mapping) was originated by Halpern [9, 10] where he obtained a generalization of the Schaunder - Techonov theorem. There is a long list of workers related to our topic of research.

Defenitions:-

1.Topology and topological Spaces

Let X be a non-empty set. Consider the family J of Subset of X satisfying the following Conditions.

(1) $\emptyset \in J$, $X \in J$

(ii) Arbitrary union of members of J is again a member of J,

i.e , If $\{G_i)\ i \in I$ be any arbitrary family of members of J, then $\cup G_i \in J$
$i \in I$

(iii) Intersection of any two members of J is again a member of J.

i.e, if G_1 and $G_2 \in J$ be any two members of $G_1 \cap G_2 \in J$. In this case, the family J in called topology on X and the pair (X,J) & Called topological space.

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2. Open Set

Let (X , J) be any topological space where J is a topology on X, then the members of J are called open set in X.

Remarks

In terms of open set.

(i) $\emptyset$ and X are open set in X

(ii) Arbitrary union of open sets in X is open.

(iii) Intersection of any two open sets in X is open.

3. Closed set

Let (X, J) be any topological space where J is a topology on X then a subset A of X is Called closed in X if its Complement A' of X is called closed in X if its complement $A' \in J$

Hence A in closed in X $\Rightarrow A' \in J$

Thus the Complement of the members of topology J is Closed in X.

4. Metric Space

Let X be any non-empty set and Consider a mapping $d: X \times X \to R$ which satisfying the following Conditions.

(i) $d(x , y) \geq 0 \ \forall$, $\mathbf{x , y} \in X$

(ii) $d(x,y) = 0 \Leftrightarrow x = y$

(iii) $d(x,y) = d(y,x) , \forall \ \mathbf{x , y} \in X$

(iv) $d(x,y) \leq d(x, z) + d(z,y) , \forall \ \mathbf{x , y,} z \in X$

In this Case 'd' is said to be metric (distance function) on the set X and the pair (x,d) is called Metric space.

5. Hausdorff space / T_2-Space

Let (X , J) be any top-space where J is a topology on X then X is said to be Hausdorff Space T_2-Space if for each pair $\mathbf{x , y} \in X$ of distinct points. There exists open set G & H such that $x \in G$, $y \in H$ and $G \cap H = \emptyset$

6. Continuity in a Topological space

Let (X , J) and (Y, J) be any two topological spaces then a mapping $f : X \to Y$ is said to be a Continuous at a point $x \in X$ if for open set V Containing f(x) there exists a open set U containing x in X Such that $\boxed{f(U) \subseteq V}$. Moreover f is said to be Continuous in X if f is Continuous at every point of X.

7. Normed Linear Space

Let X be any vector space (Linear space), then X is said to be normed linear space (denoted as $\| \ \|$) if it satisfying the following Conditions.

(i) $\| x \| \geq 0 \ \forall \ x \in X$

(ii) $\| x \| = 0 \Leftrightarrow x = 0$

(iii) $\| \alpha x \| = |\alpha| \ \| x \| \ \forall \ x \in X$ and $\alpha \in F$

(iv) $\| x + y \| \leq \| x \| + \| y \|$ (Triangle inequality)

In this Case the pair [x, $\| \ \|$] is called normed linear space.

Note: If the norm Satisfying the Conditions (iii) & (iv) then it is called Semi–norm .

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8 .Convergent Sequence in a normed linear space.

Let (x, || ||) be a normed linear-space, then Sequence $< x_m >$ in X is said to be Converge to an element $x_o \in X$, if for given $\in > 0$, then exists a positive integer m_o such that $\| x_n - x_0 \| < \in$, $\forall\ n \geq m_o$

If X_n Converges to X_0 we can write $X_n = X_0$ or $X_n \rightarrow X_0$ as $n \rightarrow \infty$

Hence if $X_n \rightarrow X_0$ iff $\| X_n - X_0 \| \rightarrow 0$

9. Cauchy Sequence in Normed linear space.

Let (X, || ||) be a normed linear space, then a sequence $< X_n >$ in X is called Cauchy Sequence if for a given $\in > 0$, there exists a positive integer m_o such that $\| X_m - X_n \| < \in$, $\forall$ m, n $\geq m_0$

10. Complete Metric space.

Let (X, || ||) be a normed linear space, then X is called complete metric space it every Cauchy sequence in X is Convergent in X. In other words.

A metric space (X, d) in which every Cauchy Sequence is convergent is said to be complete metric space.

11.Banach space

A normed linear space which is complete as a metric space is called Banach space.

or

A normed linear space in which every Cauchy sequence is Convergent is called Banach space.

12. Hilbert space

A complex Banach space H is Called Hilbert Space if a complex number (x,y) is called the inner product of X and Y is satisfysing the following properties.

(i) (x,y) = (y, x)

(ii)$(\alpha x + \beta y , z) = \alpha (x,z) + \beta (y,z)$

(iii) $(x , x) = \| x \|^2$, $\forall$ x,y,z $\in$ H and $\alpha , \beta \in$ F.

13. Connected Space

Let X be a topological space. A Separation of X is a pair U,V of disjoint non-empty open Sub-sets of X whose union is X. The space X is said to be connected if there does not exist a Separation of X.

or

A topological space X is said to be connected iff the only subsets of X that are both open and closed in X are the empty set and X itself.

14. Covering of Top space

A Collection A of Subsets of space X is said to cover X, or to be covering of X, if the union of the elements of A is equal to X. It is called an open covering of X if its elements are open subsets of X.

15. Compact space.

A space X is said to be Compact if every open covering A of X contains a finite sub collection that also covers X.

16. Definition:-

Let X be a Banach space and T be a mapping of X into itself satisfying.

$\| T_x - T_y \| \leq \| x-y \|$ for all x,y in X. Then T is said to be non-expansive.

In fact fixed point theory plays a dominent role in the field of Economics, Game theory, Physics etc. to solve different equations in these fields.

2.Hypothesis:-

The proposed work will include study of some fixed point theorems in Banach space and other spaces by using non- expansive mappings, Contraction type mappings along with Weakly Compatible mappings in Cone Banach space. we will also establish Some fixed point theorems in generalized metric spaces. we will use basic definitions, principles and theorems already established and trusted by many workers in the field.

3. Purpose and Need of the Study

The aim and main purpose of our study is to provide some new type of Contraction mappings and apply the same to prove some fixed point results.

- To introduce some new type of expansive and non-expansive mappings, Compatible mappings and utilize the same to prove some fixed point results in Banach and other spaces.
- To derive some fixed point results in Setting of Some Banach spaces and some generalized spaces.
- To explore the possibility of applications to our newly proved results.

our study undertaken is much needed, for deeper and wider vision in functional analysis, and specially in fixed point theory.
Further our investigation is very useful in Gauce theory, Economics, and SocialScience etc.

4. Methodology:-

while preparing the undertaken Project we shall various relavant definitions, propositions, theorems and Corollaries already established by previous workers using different journals, periodicals, reference books and net working.

5.Work Plan (Chapterisation)

The whole of our Studied will be divided into the following chapters.

Chapter-I :-Introduction (with literature review)
Chapter-II :-Some Common fixed point theorems and non-expansive mappings in Banach space
Chapler-III :-Fixed point theorems and Contraction mappings.
Chapter-IV :-Approximation of fixed points for Suzuki's generalized Contraction type mapping in Banach space.

Chapter-V :- Common fixed point theorems for weakly Compitable mappings in Cone Banach spaces
Chapter-VI :- Some fixed point theorems in generalized methic spaces.

Changes in some chapters will be done if needed while preparing the theses.

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28/11/25

Signature of Supervisor
(Dr. Ramendra Kumar)

Mukesh Kumar

Signature of the Candidate

UNIVERSITY DEPTT. OF H.O.D

01-12-25

Signature Of HOD
University Department Of Mathematics